# Small Filters Based on Slotted Cylindrical Ring Resonators

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*Abstract* **The realization of a small four-pole Chebyshev filter using four matched slotted cylindrical ring (SCR) resonators, is explained. This filter, which does not need tuning screws, is low-cost to manufacture and suitable for mobile communications. The filter has a bandwidth of 75 MHz, centered at 1.73 GHz. To obtain the coupling distances between the resonators, a new technique based on the quasi-magneto-static finite-difference method, is also reported.**

## I. INTRODUCTION

Very recently, the slotted cylindrical ring (SCR) was considered by some filter manufacturers as a promising resonator for developing small filters for mobile communications. This resonator has a compact structure, capable of providing medium to high Q-factors, and can provide large frequency separation between the resonnant frequencies of the fundamental and the higher order modes.

The structure of the resonator is shown in Fig. 1.a. It is a hollow conductor cylindrical ring with a slot along its length. In this resonator, the ring is equivalent to the inductor and the slot to the capacitor. When the length and diameter of the ring are much smaller than the wavelength at the operating frequency, the fundamental resonance and the first higher order resonance are largely separated in frequency. This leads to large spurious-free out-of-band response for a filter using such resonators. As an example, Fig. 2 shows the separation between the fundamental and first higher order mode of a small isolated resonator (ie: without enclosure) with the fundamental resonant frequency at 1.690 GHz. The result is obtained using a commercial TLM software.

The design of a filter based on the SCR resonator, requires an accurate knowledge of the resonant frequency and coupling coefficients. There is very limited information on the characteristics of single or coupled SCR resonators. Very recently, we have introduced a fast, accurate technique for the computation of the resonant frequency of the SCR resonator [1]. The technique has several advantages over the previous approximate closed-form expressions reported in [2, 4] for the calculation of the inductance (L), the capacitance (C), and the Q-factor of the SCR resonator. As far as the coupling between two

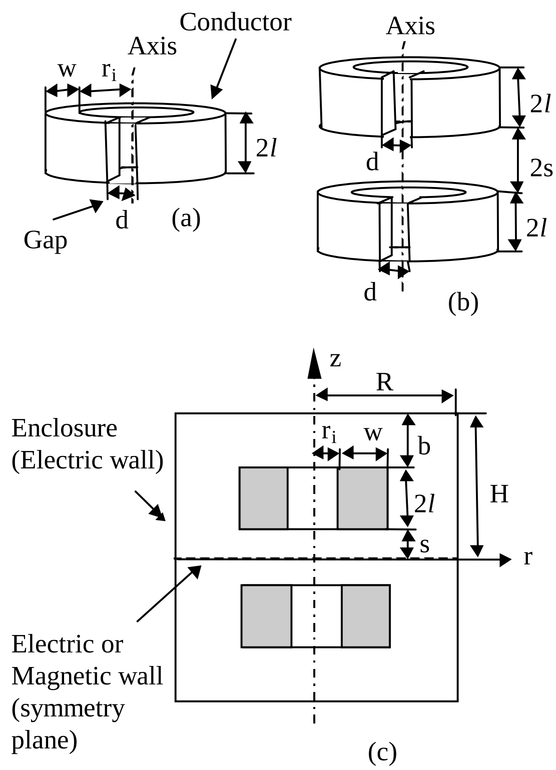


Fig. 1. (a) Slotted cylindrical ring resonator, (b) coupled resonators, and (c) coupled cylindrical rings

SCR resonators is concerned, the literature reveals only a set of experimental coupling coefficients, which are for some specific resonators [3, 4].

This paper initially reports a new technique using quasi-magneto-static assumption and the finite-difference method for the computation of the coupling coefficients between two SCR resonators positioned longitudinally, Fig. 1.b. It, then, explains the design of a four-pole Chebyshev filter with 75 MHz bandwidth and 1.73 GHz center frequency. Besides its compact size, the filter has no tuning screw. This fact supports the good accuracy of the new technique in predicting the coupling coefficients

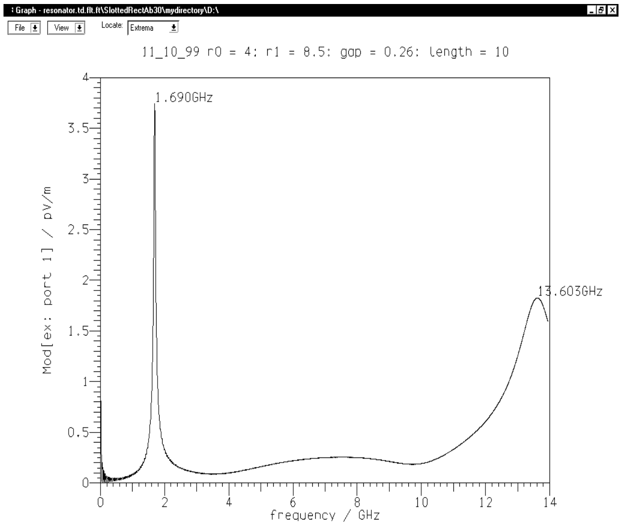


Fig. 2. The TLM computed fundamental and first higher order resonant frequencies of an isolated SCR resonator; $2l$=10 mm, $r_i$=4 mm, w=4.5 mm, and d=0.26 mm.

as well as the low sensitivity of the filter structure to inevitable small manufacturing errors.

## II. Coupling Analysis

The structure of two coupled SCR resonators arranged longitudinally, is shown in Fig. 1.b. Since the slot is very narrow, the coupling between the two resonators is predominantly magnetic and hence, they can be approximated by the magnetic coupling between two current carrying cylindrical rings of the same dimensions as the resonators, Fig. 1.c. Since in practice the filter structure is within an enclosure, the analysis assumes that the resonators are positioned symmetrically within a conductor cylindrical enclosure.

The coupling coefficient, k, can be obtained from the expression

$$k = \frac{L_m - L_e}{L_m + L_e} \tag{1}$$

where $L_m$ is the even mode inductance and $L_e$ is the odd mode inductance associated with the two cylindrical rings. Expression (1) is similar to the expression given in [5] for the calculation of the coupling between two identical coupled resonators.

The even or odd mode inductance of the structure in Fig. 1.b, can be evaluated by considering that one of the rings while the symmetry plane between the two resonators is replaced by the magnetic or electric wall respectively, Fig. 1.c. In other words, the coupled resonator problem is reduced to a single resonator problem with appropriate boundary conditions.

The problem in Fig. 1.c is solved using the stream function approach in the cylindrical coordinate system. In this approach the function that generates the field lines are obtained. Assuming that $\Phi$ is the function, it is related to the magnetic field components produced by the current over the ring as follows:

$$H_r = \frac{1}{r}\frac{\partial \Phi}{\partial z} \tag{2.a}$$

$$H_z = -\frac{1}{r}\frac{\partial \Phi}{\partial r} \tag{2.b}$$

where $\Phi$, due to the angular symmetry of the structure, is independent of $\phi$ (the angle in the cylindrical coordinate system). It is not difficult to show that $\Phi$ satisfies the following differential equation

$$\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{3}$$

which, in this work, was solved using the five-point finite-difference method [1]. The evaluation of the even and odd mode inductances are straightforward after solving (3).

The results in Fig. 3 shows the coupling coefficients versus the distance, 2s, between two coupled cylindrical rings with dimensions $2l$=10 mm, $r_i$=4 mm, w=4.5 mm. If a slot with d=0.26 mm is introduced along the ring and the ring is enclosed in an enclosure, the resulting resonator resonates at 1.716 GHz and have a Q-factor of about 2000

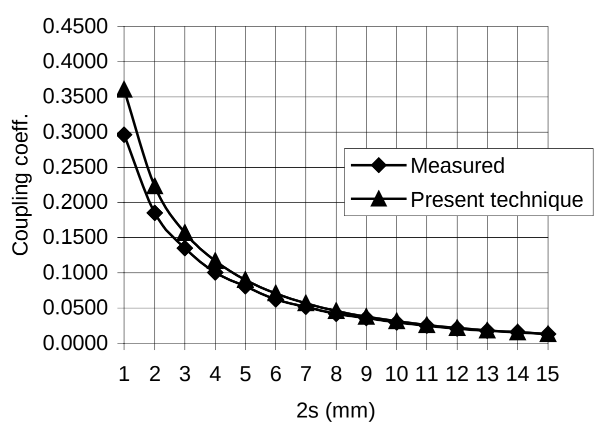


Fig. 3. Coupling coefficients versus the separation between the resonators; $2l$=10 mm, $r_i$=4 mm, and w=4.5 mm.

when silver plated. The Q-factor drops to approximately 1300 if the resonator is made in aluminum.

As Fig. 3 shows, the coupling is extremely high when the distance between the two resonators are very small. The measured coupling coefficients show lower values than the predicted ones by as much as 20% for the tight spacing of 2s=1 mm. This can be attributed to the effect of electric coupling between the two slots in the theoretical analysis. However, such large couplings are not required in the design of narrow-band filters. As seen in Fig. 3, the measured coupling coefficients approach the predicted values as the spacing between the two resonators, increases.

## III. FILTER

The filter is specified to be four-pole Chebyshev filter with center frequency at 1.73 GHz, 75 MHz bandwidth, 0.1 dB ripple level, and matched to 50 ohms at the input and output. In this case, from [6], the coupling coefficients were determined to be k12=k34=0.036025 and k23=0.02851.

Four matched aluminum resonators with all the dimensions the same as those specified for obtaining the coupling data in Fig. 3, are employed to fabricate the filter. For these resonators, d=0.26 mm, leading to the resonant frequency of about 1.716 GHz. Manufacturing highly matched SCR resonators is easily possible if correct cutting technique is employed. Experience shows that resonators with slightly (ie: about 1%) lower resonant frequency is required in order to achieve the desired center frequency.

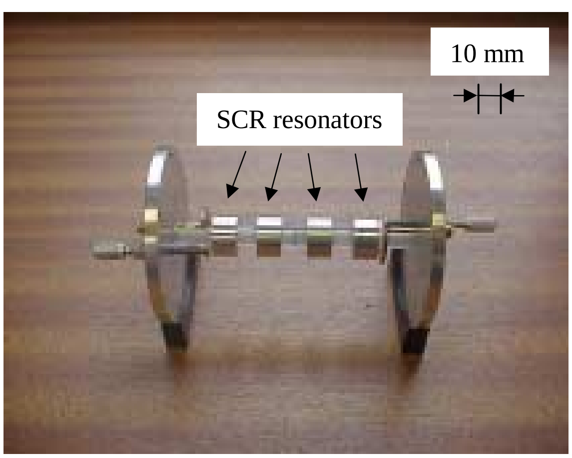


Fig. 4. The structure of the filter.

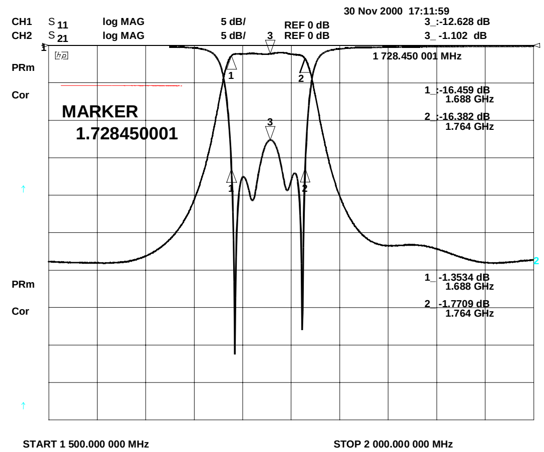


Fig. 5. The filter response.

From Fig. 3, separation distance 2s=8.96 mm between the first and second resonators and between the third and the fourth resonators, and 2s=10.12 mm between the second and the third resonators, are required to realize the coupling coefficients. For this filter, external Q-factor Qe=25.58, which has been achieved by arranging horseshoe capacitive couplings at the input and output of the filter.

The filer structure is shown in Fig. 4 and its response in Fig. 5. The filter bandwidth is correctly realized and its insertion-loss is about 1 dB. From the response, it can be infer that there is some fixed coupling between the input and the output probes. This could be due to the enclosure. By changing the size of the enclosure, this spurious coupling is expected to vanish. By silver-plating the resonators, their unloaded Q-factor increases from 1300 to about 2000, leading to a substantially lower insertion-loss. The length of this compact filter is about 100 mm.

## IV. CONCLUSION

For the theoretical prediction of the coupling between two SCR resonators positioned longitudinally, a new quasi-magneto-static technique was reported. Using a set of matched aluminum resonators, a fourth-order Chebyshev filter was designed, fabricated and tested. This compact filter, which is about 100 mm long, utilizes resonators with maximum dimension of 10 mm and does not require tuning. The insertion-loss of the filter is about 1 dB, which is expected to improve substantially when aluminum resonators are replaced by silver-plated resonators.

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